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Experimental momentum spectra of identified hadrons at e^+e^- colliders compared to QCD calculations

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Abstract

Experimental data on the shape of hadronic momentum spectra are compared to theoretical predictions in the context of calculations in the Modified Leading Log Approximation (MLLA), under the assumption of Local Parton Hadron Duality (LPHD).

Considered are experimental measurements at e^+e^- -colliders of ξ_p^* , the position of the maximum in the distribution of $\xi_p = \log(1/x_p)$, where $x_p = p/p_{\text{beam}}$. The parameter ξ_p^* is determined for various hadrons at various centre of mass energies. The dependence on the hadron type poses some interesting questions about the process of hadron-formation. The dependence of ξ_p^* on the centre of mass energy is seen to be described adequately by perturbation theory. A quantitative check of LPHD + MLLA is possible by extracting a value of α_s from an overall fit to the scaling behaviour of ξ_p^* .

1 Introduction

During a few years of LEP running, a large amount of information has been collected on identified hadron species in jets. The higher centre of mass energy of LEP, compared to past e^+e^- accelerators, makes it easier to separate the behaviour of hadrons with a high momentum, which are correlated strongly to the primary quark, from those with a low momentum, created mainly during fragmentation and hadronisation.

Clear scaling violations have been observed in the shape of the charged particle distribution of $x_p = p/E_{\text{beam}}$ as a function of the centre of mass energy. The strong coupling constant has been extracted from these scaling violations, using the behaviour at high momenta: $0.2 < x_p < 0.7$ [1]. Due to the larger statistical errors this is not possible for individually identified hadron species.

However, low momentum data for specific types of hadrons may be used to study the properties of jet-evolution and hadron-formation in the context of the LPHD hypothesis and MLLA calculations of parton spectra [2,3,4,5]. The assumption of ‘Local Parton Hadron Duality’ (LPHD) states that a calculated spectrum for ‘partons’ in a ‘parton shower’ can be related to the spectrum of real hadrons by simple normalisation constants. These constants have to be determined by experiment. A second assumption is that the low momentum part of the spectrum is not influenced in a significant way by hadrons that are correlated to the primary quark.

Calculations of the parton spectra in the ‘Modified Leading Log Approximation’ (MLLA) take into account next-to-leading logarithms in a consistent fashion. The physical mechanism relevant to these next-to-leading logarithms is the coherent emission of soft gluons inside a jet, leading to an angular ordering and an effective transverse momentum cutoff for the partons. Parton jets develop through repeated parton splittings, resulting in an increase of the multiplicity at lower momenta. The interplay of coherent emission of gluons and the creation of hadrons causes this spectrum to be cut off at very low momenta. Calculations predict the shape of the distribution of $\xi_p = \log(1/x_p)$. The resulting ‘hump-backed’ distribution is nearly gaussian. As an example, the Monte Carlo spectrum of the Λ baryon in Z^0 decays can be seen in figures 1 and 2. In the following, the maximum ξ_p^* in the ξ_p distribution will be determined for various types of hadrons and at various centre of mass energies. Subsequently a comparison is made with the theoretical calculations.

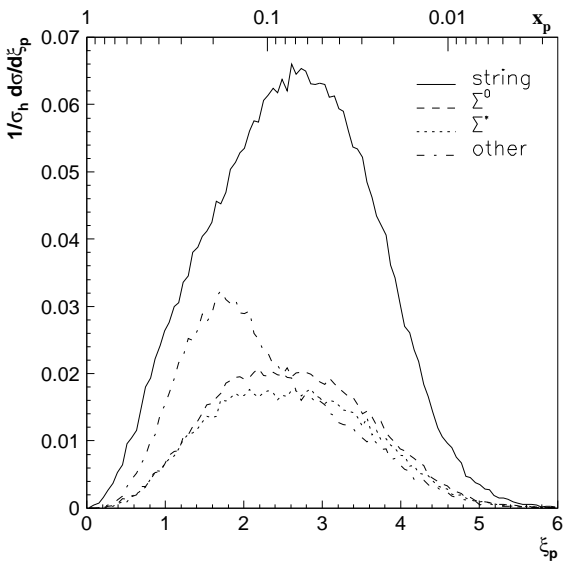


Figure 1: The dependence of the Λ spectrum on the species of its ‘parent’ particle in Z^0 decays simulated by JETSET. Particles produced directly in the fragmentation process have the Lund-model string as ‘parent’. The scales of both x_p and ξ_p are given for comparison.

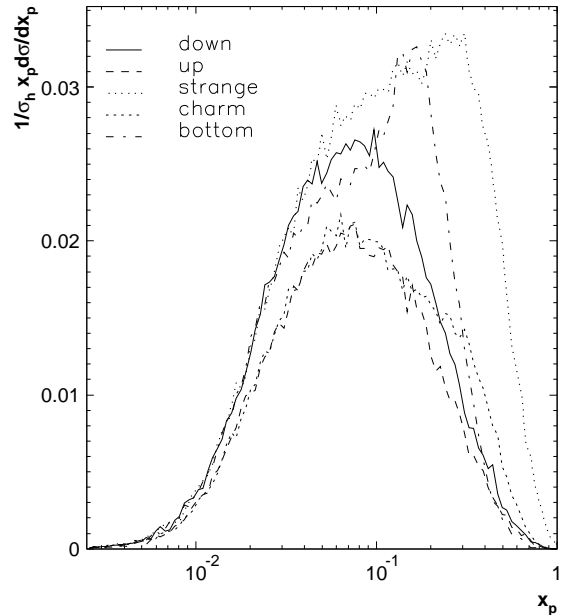


Figure 2: The dependence of the Λ spectrum on the flavour of the primary quarks in Z^0 decays at LEP, as predicted by JETSET. Note that there is not a clear separation into a region of high momentum (correlated to primary quarks) and another of low momentum (related to jet fragmentation).

	\sqrt{s}	charged	π^\pm	π^0	K^\pm	K^0	p	Λ	Ξ^-
ARGUS	9.98		2.32 ± 0.022	2.42 ± 0.08	1.64 ± 0.05	1.72 ± 0.04	1.63 ± 0.07	1.44 ± 0.034	1.32 ± 0.11
CLEO	10.49		2.30 ± 0.08			1.68 ± 0.17	1.67 ± 0.10	1.47 ± 0.08	
TASSO	14	2.45 ± 0.05	2.66 ± 0.06		1.91 ± 0.13	1.65 ± 0.13	1.80 ± 0.15		
JADE	14			2.68 ± 0.27					
TASSO	22	2.74 ± 0.06	2.99 ± 0.09		2.41 ± 0.23	2.47 ± 0.8	2.14 ± 0.27	1.75 ± 0.50	
JADE	22.5			2.84 ± 0.30					
HRS	29		> 3.3		2.28 ± 0.42	2.25 ± 0.30		1.96 ± 0.12	
TPC/ 2γ	29		3.00 ± 0.05		2.14 ± 0.05	1.98 ± 0.09	2.13 ± 0.11		
TASSO	34	3.00 ± 0.05	3.17 ± 0.05		2.35 ± 0.20	2.32 ± 0.10	2.24 ± 0.09	2.15 ± 0.15	
JADE	35			3.24 ± 0.05					
TASSO	44		3.37 ± 0.07		< 3.5		< 3.5		
JADE	44			3.52 ± 0.08					
TOPAZ	58	3.42 ± 0.04	3.51 ± 0.07		2.83 ± 0.10		2.64 ± 0.06		
OPAL	91.2	3.60 ± 0.04	3.79 ± 0.04		2.73 ± 0.06	2.87 ± 0.07	2.95 ± 0.09	2.78 ± 0.08	2.55 ± 0.20
DELPHI	91.2					2.62 ± 0.11		2.81 ± 0.04	
L3	91.2	3.71 ± 0.05		3.96 ± 0.13		2.89 ± 0.05			

Table 1: The values of ξ_p^* for various hadrons as determined from the momentum spectra as measured by e^+e^- experiments at different values of \sqrt{s} . For the η meson, the L3 experiment reported a value $\xi_p^* = 2.60 \pm 0.15$.

Figure 1 shows that LPHD is not an obvious assumption: in the JETSET Monte Carlo [6], the spectrum of e.g. a Λ baryon depends on its ‘parent’ particle. The LPHD assumption states that the sum of these spectra is proportional to the spectrum as calculated for a parton shower with a correctly chosen energy cut-off.

Moreover, the JETSET momentum spectrum of the Λ baryon (figure 2) shows a strong dependence on the flavour of the primary quarks. For the heavier quarks, s, c and b, the flavour dependence does not only manifest itself at high momenta, but also affects the distribution at low momenta. This will cause shifts of ξ_p^* that depend on the jet-flavour.

The spectrum of all charged particles at LEP has been fitted to the MLLA distribution, with free normalisation factors for pions, kaons and protons, depending on the centre of mass energy [7,8]. It was quite surprising [4] that the MLLA functions can also fit the high momentum part in the data for pions. This is probably a coincidence, since the heavier K^0 meson has a spectrum that can not be fitted as nicely at large x_p .

One way to see whether the observed shape of the spectrum is really due to coherent gluon emission is to compare experimental data to predictions of the JETSET Monte Carlo program, with the coherence (angular ordering) either turned on or turned off. The large dependence on the primary quark flavours and the fact that coherence in JETSET is not really necessary to fit the experimental data has led some to the conclusion [9] that coherence can not

be demonstrated on the basis of hadron momentum spectra.

Here an attempt will be made to infer more information using the properties of identified hadrons, concentrating on the observable ξ_p^* . This is done for data from LEP and from various other e^+e^- colliders. Subsequently, the available results for ξ_p^* are compared to the LPHD + MLLA approach [2,3].

2 Experimental values of ξ_p^* for identified hadrons

At LEP, analyses of explicitly identified hadrons have been performed for the π^\pm [10], the π^0 [8,11], the K^\pm [10], the K_S^0 [11,12,13], the η [14,11] the proton [10] and the Λ [12,15,16,17]. At lower statistics data is also available for the Ξ^- and Ω^- baryons, as well as the Σ^* and Ξ^* [12,15].

The values of ξ_p^* correspond to low momenta, where the dependence on the primary quark flavours is expected to be small. To first approximation the distribution in ξ_p is gaussian, but a distorted gaussian fits the calculated spectrum more accurately [18,3]. The statistics of identified hadrons is not sufficient to extract the width and the two distortion parameters for more than a few experiments.

However, since these other parameters depend more strongly on the high momentum tail of the distribution, they can be expected to depend more strongly on the event flavour, which could bias

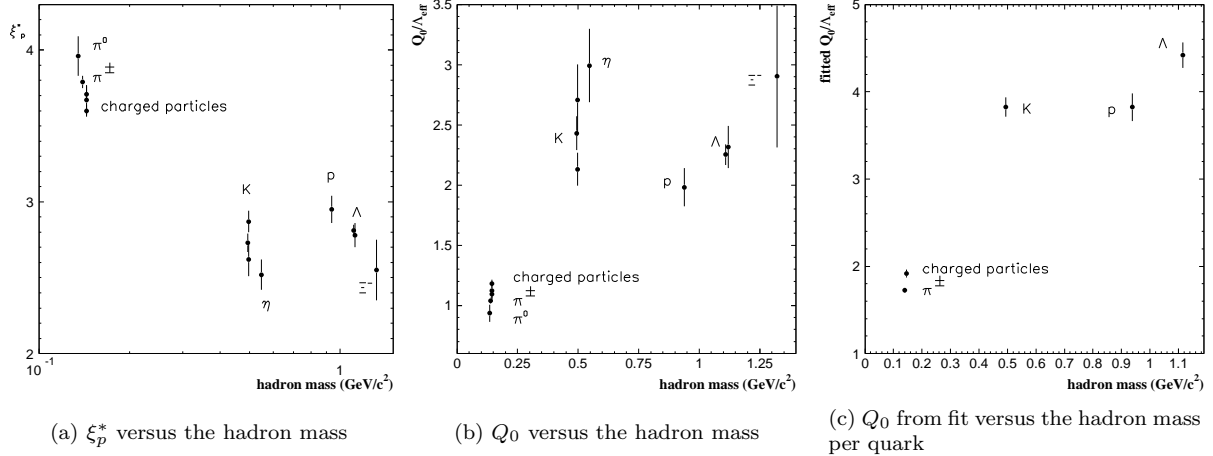


Figure 3: The maxima ξ_p^* of the distributions in $\xi = -\log x_p$ for different hadrons as measured at LEP (table 1) are shown in figure (a). Two different determinations of the corresponding scale Q_0 are shown in figures (b) and (c). The calculated dependence for $\Lambda_{\text{eff}} = 50$ MeV leads to figure (b), where one would naively expect Q_0 to be proportional to the hadron mass. Figure (c) gives the values of Q_0 as determined from the fit of the dependence of ξ_p^* on E_{beam} . A (large) common error on the different values of Q_0 has been neglected in both figures (b) and (c). This causes the vertical scales to be rather arbitrary, but the relative position of the points remains significant.

the results. We will suppose that this violation of LPHD can be avoided adequately by concentrating on ξ_p^* , a property of the low-momentum part of the spectrum.

In this paper ξ_p^* is simply defined as the maximum of the distribution. The values were determined by fitting the ξ_p distribution in a limited range ($\approx \pm 1$) of ξ_p around the maximum, to a gaussian distribution. A systematic error was estimated by changing the fitted range and by fitting the same range to a polynomial function. The statistical and systematic errors were added in quadrature. The systematic error often dominated.

For the most accurately determined experimental spectra, those of identified charged hadrons at OPAL [10], a more complicated function is necessary. In this case a polynomial of order 4 or 5 was used to fit either the partial cross section, or its logarithm¹. These polynomials were defined such that the maximum was one of the free parameters in the fit. In this way the MINOS procedure of MINUIT [27] can automatically determine an interpolation error due to variations of the other free parameters of the polynomial. The values found for the maxima in OPAL's spectra are different from those reported by OPAL, and the estimated errors are slightly larger.

¹Fitting the logarithm to a 4th order polynomial is equivalent to using a distorted gaussian.

The results are given in table 1. Use was made of spectra published by CLEO [19], ARGUS [20], TPC [21], HRS [22], TASSO [23], JADE [24], TOPAZ [25], DELPHI [12,17], OPAL [7,13,15,10] and L3 [8,14,11]. Most of the recent publications reported values of ξ_p^* , but they did not all use exactly the same definition of it. While the numbers given by L3 were compatible with the method used here, OPAL and DELPHI used slightly different definitions and for consistency the values of ξ_p^* are those determined by the author from the published experimental spectra. The spectra from TOPAZ were not available in [25], only values of ξ_p^* ; likewise for the recent paper by L3 [11]. For most experiments at lower centre of mass energies the authors did not report the distribution of ξ_p or x_p , but used the energy to define $x_E = 2E/E_{\text{beam}}$, or they divided by $\beta = v/c$ to obtain the 'scaling cross section'. Many different normalisations have been used, but fortunately this is unimportant for a determination ξ_p^* . From the experiments (ARGUS and CLEO) near the Υ resonances results are given for the 'continuum', and for the resonances itself, where the contributions from the continuum are subtracted. The values in table 1 refer to data from the continuum above the Υ resonance. In the data from ARGUS for charged pions and protons, the contributions from decays of K_S^0 and Λ were subtracted.

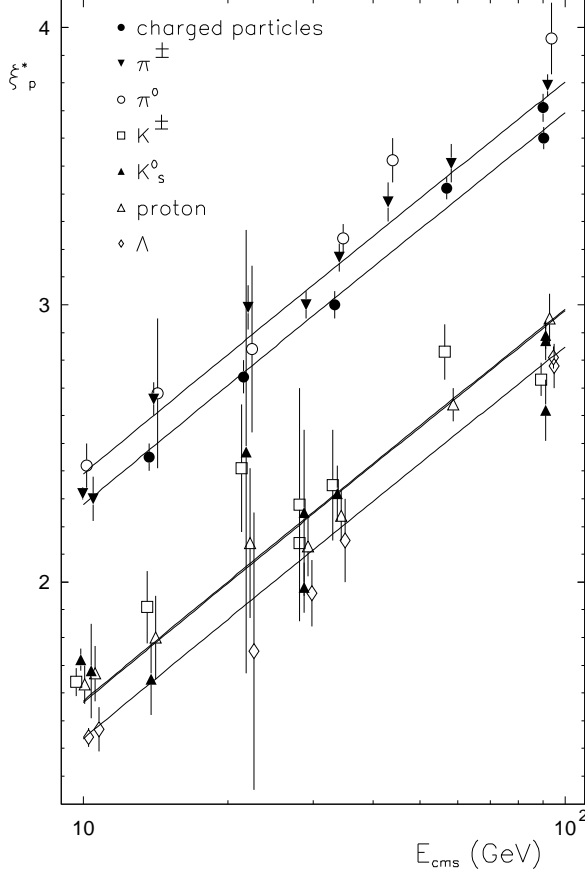


Figure 4: The experimental values of ξ_p^* versus \sqrt{s} for various identified hadrons. The curves are from the combined fit of Λ_{eff} and $F(\text{hadron})$ to the predicted dependence (equations (1) and (11)). From top to bottom, the curves correspond to pions (π^\pm , π^0), all charged particles, kaons (K^\pm , K^0), protons and Λ baryons. The curves for kaons and protons happen to lie on top of each other.

3 Predictions for ξ_p^*

The LPHD + MLLA calculations [3] of the distribution of ξ_p depend on an effective QCD scale $\Lambda_{\text{QCD}} \sim \Lambda_{\text{eff}}$ and on a transverse momentum cutoff Q_0 in the evolution of the parton cascade. The value of ξ_p^* is calculated to be a nearly linear function of $\log Q_0$, $\log \Lambda_{\text{eff}}$ and $\log E_{\text{beam}}$.

The calculated dependence on the centre of mass energy is as follows [3]:

$$\xi_p^* = Y \left[\frac{1}{2} + \sqrt{C/Y} - C/Y + \mathcal{O}(Y^{-3/2}) \right] + F(\lambda), \quad (1)$$

where $F(0) = 0$ and ²

$$Y = \log(E_{\text{beam}}/\Lambda_{\text{eff}}), \quad \lambda = \log(Q_0/\Lambda_{\text{eff}}), \quad (2)$$

and Λ_{eff} is an effective QCD scale, while only the cutoff scale Q_0 depends on the hadron type. The constant C is calculated to be

$$C = \left(\frac{a}{4N_c} \right)^2 \frac{N_c}{b}, \quad (3)$$

where

$$a = 11N_c/3 + 2n_f/(3N_c^2), \quad (4)$$

$$b = 11N_c/3 - 2n_f/3. \quad (5)$$

$N_c = 3$ is the number of colours and $n_f = 3$ is the active number of quark flavours in the fragmentation process. For $n_f = 3$ one finds $C = 0.2915$.

It is important to note that F does not depend on Y , and that the first part of equation (1) is independent of Q_0 . This predicted behaviour can be checked by comparing spectra of different identified particles. In the available momentum range this leads to a nearly linear dependence of ξ_p^* on Y .

In ref. [3] various graphs show the results of numerical calculations for the dependence of various parameters on E_{beam} and $Q_0 \neq \Lambda_{\text{eff}}$. Figure 4 of ref. [3] shows the dependence of ξ_p^* on Q_0 for $\Lambda_{\text{eff}} = 150$ MeV, and at a number of beam energies. From this it is possible to extract F after subtracting the Y dependent part of equation (1). For an interpretation of the experimental values of ξ_p^* it was useful to fit both this function $F(\lambda)$, and its inverse $\lambda(F)$ to polynomials. The result for $F(\lambda)$ is:

$$F(\lambda) = -1.46 \cdot \lambda + 0.207 \cdot \lambda^2 \pm 0.06 \quad (6)$$

while the inverse of F is described by:

$$\lambda(F) = -0.614 \cdot F + 0.153 \cdot F^2 \pm 0.06 \quad (7)$$

The given errors denote the maximum deviation from the distribution plotted in ref. [3], in the ranges $-2 < F < 0$ and $0 < \lambda < 2$.

4 The meaning of Λ_{eff}

The QCD scale Λ_{eff} [26] is related to the (running) strong coupling constant by:

$$\frac{\alpha_s}{2\pi} = \frac{1}{bY} = \frac{1}{b \log(E_{\text{beam}}/\Lambda_{\text{eff}})}. \quad (8)$$

²This detail [26] is not completely clear in reference [3], where equation (1) is given as a function of $Y - \lambda$, but in the approximation that $\lambda = 0$.

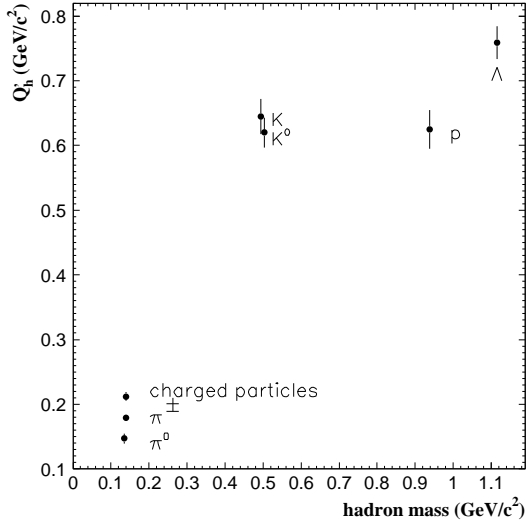


Figure 5: The values of Q'_0 from table 2 versus the mass of the hadron.

This makes it possible to express the scale dependence of eq. (1) in terms of α_s :

$$\frac{\partial \xi_p^*}{\partial Y} = \frac{1}{2} + \frac{1}{8}a\sqrt{\frac{\alpha_s}{2\pi N_c}} + \mathcal{O}(\alpha_s^{5/2}), \quad (9)$$

$$\frac{\partial^2 \xi_p^*}{\partial Y^2} = \frac{-ab}{16\sqrt{N_c}} \left(\frac{\alpha_s}{2\pi}\right)^{3/2} + \mathcal{O}(\alpha_s^{7/2}). \quad (10)$$

The $\sqrt{\alpha_s}$ term is the next-to-leading (MLLA) correction to the leading term. The next-to-next-to-leading term $\mathcal{O}(\alpha_s)$ vanishes [26].

Although this sounds impressive, it is still not enough for an unambiguous definition of Λ_{eff} or, equivalently, of $\alpha_s(E)$ at a well-defined energy scale E . An uncertainty of e.g. a factor 2 in the energy scale leads to $\alpha_s(2E)$ instead of $\alpha_s(E)$. This induces a correction in equation (9) of $\mathcal{O}(\alpha_s^{3/2}(E))$, which shows that for a better definition of the energy scale it is necessary to know the $\mathcal{O}(\alpha_s^{3/2}(E))$ correction. This is the reason for talking about the scale Λ_{eff} instead of a more well-defined scale like $\Lambda_{\overline{\text{MS}}}$.

If we now neglect the dependence of $\alpha_s(E)$ on E , ξ_p^* becomes a linear function of $\log(E_{\text{beam}})$, where the slope is given by equation (9).

5 Dependence of ξ_p^* on \sqrt{s}

A more quantitative comparison is possible for the scaling behaviour of ξ_p^* , as parametrised by the effective QCD scale Λ_{eff} . The dependence on $\sqrt{s} = 2E_{\text{beam}}$ of the experimental values of ξ_p^* from table 1 is presented in figure 4.

As a start, the experimental values can be fitted to separate first- or second order polynomials in $\log E_{\text{beam}}$ for each type of particle. The quality of these fits provides an internal check of the estimated experimental errors. Table 2 gives the slopes of fitted straight lines and the values of χ^2 for first and second order polynomials. The quality of the fits is acceptable, but the values of χ^2 when fitting so many free parameters could indicate that the experimental errors have been underestimated somewhat. It seems that the measured slopes are not quite equal to one other, as was predicted by LPHD + MLLA. This is due to some dependence on the flavour of the particles and of the primary quarks, violating the LPHD assumption slightly.

For a comparison with other publications it is also interesting to fit the data for each hadron type to equation (1), under the assumption that $Q'_0 \stackrel{\text{def}}{=} \Lambda_{\text{eff}} = Q_0$, so that $F(\lambda) = F(0) = 0$. The energy scale Q'_0 is different from Q_0 as determined in the previous section, and it is also different from Λ_{eff} . In many past publications the notation ‘ Λ_{eff} ’ is also used for Q'_0 . This is confusing, since Q'_0 is not expected to be the same for different hadrons, while the QCD scale Λ_{eff} should be independent³. The quality of this fit with $\Lambda_{\text{eff}} = Q_0 \equiv Q'_0$ is slightly worse, with an overall $\chi^2 = 2.41$. The largest contributions to χ^2 come from the kaons, that also have a different slope b_h from the other hadrons. Looking carefully at figure 4 one might also suspect the measurements done at the Z^0 and Υ .

The fitted values of Q'_0 are not equal to the corresponding values of Q_0 shown in figure 3. The values of Q'_0 are quite near to the particle mass for mesons, but for the baryons Q'_0 is significantly smaller. Figure 5 shows the dependence of Q'_0 on the hadron mass. No clear linear dependence is observed.

To check how well the data is described by the calculations, a simultaneous fit of equation (1) was done to all the data from table 1, where Λ_{eff} is a universal scale, while F is allowed to have different values for hadrons, pions, kaons and the two baryons.

Such a six parameter fit was performed by minimising χ^2 using the MINUIT [27] program, with zero as starting values for F_h . There is a strong correlation in the fit of $\log \Lambda_{\text{eff}}$ and \bar{F} , the average of F_h . Therefore it is practical to fit \bar{F} and differences $F_h - \bar{F}$. After this, the fit converges properly,

³ In principle one might expect that different hadrons from different decay chains could introduce a small dependence of an effective value of n_f on the type of hadron. However, equation (9) shows that variations of n_f from 3 to 5 can only change the slope by a few percent. This is well within the other uncertainties.

particle	a_h	b_h	$\chi^2/\text{n.d.f.}$	Q'_0	$\chi^2/\text{n.d.f.}$	points
charged	0.76 ± 0.12	0.641 ± 0.030	1.36	0.212 ± 0.007	1.10	6
π^\pm	0.79 ± 0.06	0.667 ± 0.018	1.23	0.179 ± 0.005	1.53	9
π^0	0.78 ± 0.20	0.705 ± 0.059	0.42	0.147 ± 0.008	0.70	6
π^\pm, π^0	0.78 ± 0.05	0.679 ± 0.016	1.49	0.174 ± 0.004	1.91	15
K^\pm	0.47 ± 0.11	0.512 ± 0.032	2.17	0.645 ± 0.027	4.48	8
K^0	0.49 ± 0.09	0.517 ± 0.025	2.12	0.620 ± 0.023	5.49	10
K^\pm, K^0	0.48 ± 0.06	0.516 ± 0.019	1.93	0.631 ± 0.018	4.78	18
p	0.27 ± 0.12	0.580 ± 0.036	0.37	0.625 ± 0.030	0.93	8
Λ	0.02 ± 0.07	0.613 ± 0.021	0.30	0.759 ± 0.025	1.07	7
p, Λ	0.09 ± 0.07	0.609 ± 0.019	1.29	0.712 ± 0.020	1.69	15
total $\chi^2/\text{n.d.f.}$			1.21		2.41	54

Table 2: Result of a fit of the experimental values of ξ_p^* in table 1 to a linear function $\xi_p^* = a_h + b_h \cdot \log(E_{\text{cms}}/\text{GeV})$ for various identified hadrons. The scale Q'_0 is the result of a second fit to the function $\xi_p^* = \frac{1}{2}Y + \sqrt{CY} - C$, where $Y \equiv \log(E_{\text{beam}}/Q'_0)$ and $C = 0.2915$. The fits improve slightly when data at the Υ are excluded. To compare the overall quality of the fits to the two functions, a total $\chi^2/\text{n.d.f.}$ is given, based on the fits for π^\pm , π^0 , K^\pm , K^0 , p, and Λ .

but the correlation coefficient of $\log \Lambda_{\text{eff}}$ and \bar{F} is still 100%.

The strong correlations can be understood from equation (1) and figure 4: a change in $\log \Lambda$ translates the curves in the direction of the x -axis, while a change of \bar{F} causes a translation along the y -axis. The fact that the curves are so straight causes the strong correlations. The reason that the fit still converges is that the slope of the curves varies slowly with changing Λ_{eff} .

To see if this causes numerical instability or even inaccuracy, it was subsequently checked that the correlation in the fit can be reduced to a more acceptable 74% by a change of variable to $F' = \bar{F} - r \cdot (\log \Lambda - \log \Lambda_0)$, where $r = 0.62$ is the average value of $\partial \xi_p^* / \partial Y$ as given by equation (1) and the constant $\Lambda_0 \approx \Lambda_{\text{eff}}$. The fit in this parametrisation converges to the same values of χ^2 and Λ_{eff} as when fitting \bar{F} itself.

The fit has 47 degrees of freedom and leads to $\chi^2/47 = 2.15$. This value of χ^2 should be compared to the values of the ‘total’ $\chi^2/\text{n.d.f.}$ in table 2, that are also larger than 1.

The resulting parameters and their statistical accuracy (after applying the scale factor $\sqrt{\chi^2/\text{n.d.f.}} = 1.47$ to all the experimental errors) are:

$$\begin{aligned}
\Lambda_{\text{eff}} &= 0.052^{+0.12}_{-0.044} \text{ GeV} \\
\Rightarrow \alpha_s(m_{Z^0}) &= 0.103 \pm 0.022 \\
\bar{F} &= -1.29^{+0.75}_{-1.1} \\
F_\pi - \bar{F} &= 0.54 \pm 0.021 \\
F_{\text{charged}} - \bar{F} &= 0.42 \pm 0.027 \\
F_K - \bar{F} &= -0.28 \pm 0.027 \\
F_p - \bar{F} &= -0.28 \pm 0.039 \\
F_\Lambda - \bar{F} &= -0.41 \pm 0.030
\end{aligned} \tag{11}$$

The errors here are purely statistical, as given by the MINOS method in MINUIT [27], which takes into account non-linearities and correlations between parameters. The correlation coefficient of Λ_{eff} and \bar{F} is 100%, while the values of $F_h - \bar{F}$ are constrained to have a zero sum.

Figure 4 shows that the fitted functions are very near to straight lines and that they follow the data points rather well. Addition of a free $\mathcal{O}(Y^{3/2})$ term as in equation (1) does not improve the fit significantly.

So what should one conclude from the high values of χ^2 ? It seems that the general behaviour is described well by LPHD + MLLA, but that equation (1) differs significantly from the data for each hadron. This is probably a signal of the breakdown of LPHD at this level of accuracy.

An alternative way of fitting the data is based on equation (9) with a fixed value of α_s . Allowing α_s to ‘run’ somewhat leads to the following parametrisation:

$$\begin{aligned}
\xi_p^* &= p_{0h} + p_1(\log E_{\text{beam}}/\text{GeV} - \log 15) \\
&\quad + \frac{1}{2}p_2(\log E_{\text{beam}}/\text{GeV} - \log 15)^2,
\end{aligned} \tag{12}$$

where the quadratic term is centred at 15 GeV. The constant term p_{0h} is different for each hadron type. Equations (9) and (10) relate the values of p_1 and p_2 to the strong coupling constant α_s . This fit has no problems with correlated parameters and it converges to:

$$\begin{aligned}
p_1 &= 0.613 \pm 0.015 \Rightarrow \alpha_s = 0.122^{+0.035}_{-0.030} \\
p_2 &= -0.012 \pm 0.026 \Rightarrow \alpha_s = 0.14 \pm 0.16
\end{aligned} \tag{13}$$

where α_s is given at a scale of 15 GeV. Again, the given errors are purely statistical. The value of Λ_{eff} from equation (11) gives $\alpha_s = 0.123^{+0.033}_{-0.031}$ at the scale 15 GeV, which is equal to the value derived from p_1 . Within its large error, p_2 is also consistent. The values of $\chi^2/\text{n.d.f}$ are comparable to those in the previous fit. The differences $F_h - \bar{F}$ can be determined from p_{0h} and they are consistent with those given in eq. (11).

6 Dependence of ξ_p^* on the hadron mass

The dependence of ξ_p^* on the hadron mass as determined from LEP data is shown in figure 3(a). As has been said, this dependence is not predicted by the LPHD + MLLA approach, but it is interesting to look at the relation between the scale Q_0 and the mass and flavour of the hadron. Using equations (1) and (7) and the determined value of $\Lambda_{\text{eff}} = 50$ MeV, one can convert the values of ξ_p^* (eq. 11) to values of Q_0/Λ_{eff} . The result of this is shown in figure 3(b). The error bars in this figure do not incorporate the uncertainty of Λ_{eff} . This means that the absolute scale should not be taken too seriously.

Naively one could expect that the cutoff scale Q_0 grows proportionally to the mass of the hadron. The most naive guess would be a linear dependence of the form $Q_0 \approx \Lambda_{\text{eff}} + m_{\text{hadron}}$. Figure 3(b) shows that this is not correct and that at least a separate treatment of mesons and baryons is necessary.

Figure 3(c) also shows Q_0 , but now determined from the fitted F_h in equation (11). Again the errors given do not include the very large overall error on \bar{F} and they are only significant when comparing the different points relative to each other. Taking this into account, the result is comparable to that of figure 3(b).

7 Summary

The data of LEP are giving a wealth of information about the fragmentation of quarks and gluons, as well as the mechanisms at play in the hadronisation. Although Monte Carlo models like **JETSET** and **HERWIG** can describe these data very accurately, it is a valid question whether these models with all their free parameters lead to a better understanding of the physics behind the formation of hadrons in a jet, or are merely to a better parametrisation.

The ‘LPHD + MLLA’ approach [2,3,4,5] is an interesting attempt to understand some properties of

multihadron production in jets in terms of perturbative QCD. This approach tries to take perturbative QCD as near as possible to the limits posed by confinement. It gives predictions for properties of momentum spectra that can be compared with experimental data, but concentrates on observables that can be described without too many unknown free parameters.

The experimental values of the parameter ξ_p^* for various hadrons and at various centre of mass energies was determined from the published momentum spectra. Subsequently, an investigation was made of the dependence of ξ_p^* on both the centre of mass energy and the mass of the identified hadrons, and this could be compared with the theoretical predictions.

The dependence of ξ_p^* on the mass and flavour of the identified hadron raises some interesting questions that have not yet been answered in the context of LPHD + MLLA and the description of fragmentation by truncated parton cascades.

An attempt was made to clear up some of the confusion in the definition of Λ_{eff} . The author would like to stress that it is a common but confusing practice to use the symbol Λ_{eff} for the other scale, here called Q'_0 . This is because the Q'_0 is not expected to be independent of the type of hadron, and should therefore not be interpreted as Λ_{QCD} .

The price paid for a more consistent definition of Λ_{eff} is the explicit introduction of the constants F , that are different for different hadrons. The advantage is that Λ_{eff} can now be interpreted as a QCD scale Λ_{QCD} , and that it is then possible to make a more quantitative comparison with theory.

The dependence of ξ_p^* on the centre of mass energy is described adequately by the MLLA calculations. A value of α_s has now been extracted from these data, and its value is consistent with the many accurate measurements. It is perhaps striking that data at very low momenta (at LEP, $p \approx 0.83(2.27)\text{GeV}$ when $\xi_p \approx 3(4)$), that are near to the region of phasespace where confinement occurs, can not only be described qualitatively by perturbative QCD, but can even be used to extract a consistent value of the strong coupling constant.

It is a success of LPHD + MLLA that the extracted value of α_s is correct. However, it is clear that this is not a good way to determine α_s accurately: it is nearly impossible to make a proper estimate of the systematic errors. The quality of the overall fit suggests that the picture of LPHD is starting to break down.

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